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Prediction of Two-Dimensional Baseline Warranty Service Costs Using Monte Carlo Simulation

M. Heydari[1,*]

[1] Assistant Professor, Department of Mechanical and Industrial Engineering, Faculty of Engineering, Zanjan University.

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ABSTRACT

For complex and high-value products such as automobiles, manufacturers typically provide a warranty to assure customers of the product's performance for a specific duration or level of usage. This type of contractual guarantee is often referred to in the literature as a two-dimensional baseline warranty, since it considers both calendar time and accumulated usage. During the warranty period, the manufacturer is responsible for covering the costs of service and repair when failures occur, which may impose a considerable financial burden if not properly anticipated. Therefore, accurately estimating the expected warranty cost is a critical issue for both operational planning and financial management. In this article, we address this concern by modeling the product failure process as a function of both time and usage. To capture the stochastic nature of failures, Monte Carlo simulation is employed, enabling the generation of realistic lifetime and usage scenarios. Furthermore, it is assumed that the service cost depends jointly on the age of the product at the time of failure and the usage level at that point. Based on this framework, the expected service cost over the baseline warranty period is derived. To demonstrate the practical applicability of the approach, a numerical case study is presented. The results indicate that the proposed method provides an efficient and reliable tool for predicting two-dimensional warranty costs, thereby assisting manufacturers in effective resource allocation and decision-making.

1. INTRODUCTION

Warranty is essentially a contractual guarantee between the manufacturer and the buyer regarding the sold product or services. The purpose of the warranty is to ensure the producer's responsibility for early product failures (whether physical products or services) for a specific period or usage level under predefined conditions. In this definition, failure means the product's inability to perform the expected function. In this contract, the promised performance and functionality are clearly defined, and in case of any deviation, the supplier is obligated to compensate the buyer. The buyer's responsibilities for maintaining and using the sold product are also specified in the warranty contract. This type of guarantee, which is essentially part of the product sales process and is provided

* Corresponding Author: mheydari@znu.ac.ir
Assistant Professor, Department of Mechanical and Industrial Engineering, Faculty of Engineering, Zanjan University.

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by the manufacturer without additional cost to the buyer, is called a baseline warranty. According to the U.S. Commerce Law, any product priced over $15 must come with a warranty [1].

In addition to transparency in the specifications and performance of the product during the warranty period, the process of compensating potential product failures during the warranty period should be defined, known as warranty policies. Free repair of faults, free replacement of defective parts, and the replacement of the entire product are parts of warranty policies offered for different products. However, compensating product failures during the warranty period requires time, resources, and ultimately, costs from the manufacturer. Although it can be argued that to some extent, the costs of such services are considered in the initial pricing of the product, it should be noted that ignorance of the actual service costs can result in significant financial losses for the manufacturer. Therefore, accurately estimating the service costs during the baseline warranty period is one of the important concerns of manufacturers. By knowing this cost, the manufacturer can make necessary preparations for financial provisioning and service cost management.

Research in the field of warranties supports the idea that warranty-related topics, including baseline warranties, are relatively new and have been expanding gradually for about 40 years worldwide. Currently, a considerable number of research studies have been conducted in the field of baseline warranties, covering various aspects of the subject. For further familiarity with the field of warranties, it is recommended to study references [1-5]. Among these, research conducted in the field of warranties can be classified into two categories: one-dimensional warranties and two-dimensional warranties, based on the number of variables used to define the warranty scope. Products such as televisions, refrigerators, and most electronic devices are examples of products with one-dimensional warranties. In contrast, products such as automobiles are the most recognized products with warranties defined based on time and usage levels. In these products, the baseline warranty expires after a certain period of time or a specific distance traveled, whichever occurs earlier [6]. This article focuses on moving from one-dimensional warranties to the literature of two-dimensional baseline warranties, reviewing the research in this area. For a comprehensive understanding of the conducted studies, refer to the review article.[7]

Most of the research in the field of two-dimensional warranty cost estimation has been concentrated on mathematically modeling the product failure process using stochastic processes, as indicated by studies such as [8-12]. However, it is important to note that, due to the complexity of dealing with two-dimensional stochastic processes, the two-dimensional model has been transformed into a one-dimensional model. Subsequently, the focus is on analyzing the single-dimensional model (in terms of time or usage) [13-15]. For further insight into two-dimensional modeling methods, refer to the mentioned reference [16].

On the other hand, in the analysis of the cost of these models, only the expected amount of the targeted costs has been considered, neglecting the variance or dispersion of service costs. While concentrating solely on the average warranty costs can increase decision-making risks in the realm of warranty cost management. In such cases, mathematical modeling by considering higher-order moments of the stochastic variable of cost can assist in risk management [17]. However, this leads to increased complexity in mathematical models and makes the process of understanding and implementing them more challenging. Especially with the increasing complexity of modern products, mathematical modeling, solution, and ultimately analysis are becoming progressively more difficult. In this context, employing innovative approaches and computer software can predict warranty costs with much higher speed and accuracy, and if necessary, study the effects of manufacturer interventions such as inspection, preventive maintenance, upgrades, etc.

For instance, employing simulation techniques such as Monte Carlo simulation and neural networks can contribute to estimating warranty costs [18-22]. In this article, given an understanding of the complexities of modeling and solving two-dimensional failure models, the Monte Carlo simulation is utilized to model the failure process. Subsequently, assuming that service costs are dependent on the time of failure and the product's usage at the moment of failure, the costs of product failures are estimated. Following this, the average service costs along with the extracted cost variances are presented. A histogram is then displayed, providing an appropriate visual representation for the manufacturer regarding the potential range of changes in two-dimensional base warranty costs.

The subsequent sections of the article are outlined as follows: In the next section, the steps of the Monte Carlo simulation are introduced, and the equations utilized in the simulation are explained. The third section presents a numerical example, and through the implementation of the introduced process, the estimation of failure costs is carried out. Finally, the article concludes with a summary of the findings.

2. SIMULATION OF THE FAILURE PROCESS FOR A NEW PRODUCT UNDER TWO-DIMENSIONAL BASE WARRANTY

Assuming $f(x;\alpha(r))$ is the conditional probability density function of X under the usage rate R=r, and g(r) is the probability density function of the customer's product usage rate. According to the Accelerated Failure Time (AFT) model, $\alpha(r)$ is the conditional scale parameter of X, and its value is given by $\alpha(r)=\alpha_0(r_0 / r)^\gamma$. Here, r_0 is the nominal usage rate of the customer for the product, and α_0 is the scale parameter of X under the nominal usage rate. γ is the AFT parameter, and its value is greater than one [16].

Taking the above considerations into account, the warranty expiration time will be dependent on the customer's product usage rate. This implies that for customers with a higher usage rate, the product's operational limit (denoted as *U*) will be reached sooner, leading to the completion of the warranty period. Conversely, for customers with a lower usage rate, the warranty time constraint (*W*) will be achieved earlier, resulting in warranty expiration. In other words, the warranty expiration time under the usage rate *R*=*r* is given by:

$$T_{BW|r} = \min\left\{W, \frac{U}{r}\right\} \tag{1}$$

Regarding the product failure process under the customer's product usage rate, it should be mentioned that the time to the first observed product failure is calculated within the framework of the elapsed lifetime. The product's operational performance is calculated through the following steps:

1. A random value for the customer's product usage rate is generated ($r_1 \in R$).
2. For the usage rate r_1, the value of the conditional time scale parameter until the first observed failure is calculated. This value is given by:

$$\alpha(r_1) = \alpha_0 \times \left(\frac{r_0}{r_1}\right)^\gamma$$

3. A random value for the time to the first observed failure, conditional on $R=r_1$ and assuming that its scale parameter follows $\alpha(r_1)$, is generated. This value is denoted as x_1, where:

$$x_1 \epsilon X \text{ ,} X \sim f(x; \alpha(r_1))$$

4. The operational performance of the product until time x_1, denoted as u_1, is calculated using the relationship $u_1=r_1 \times x_1$.

However, it should be noted that assuming minimal repair at the time of failure, the failure times of the product under the usage rate R=r will follow a non-homogeneous Poisson process. The simulation formula for the non-homogeneous Poisson process will be as follows [23].

The product's lifetime at the time of the first observed failure.

$$t_1(r) = \left[-\alpha(r)^\beta \times LnU_1\right]^{\frac{1}{\beta}} \tag{2}$$

The product's lifetime at the time of the second observed failure.

$$t_2(r) = \left[t_1(r)^\beta - \alpha(r)^\beta \times LnU_2\right]^{\frac{1}{\beta}} \tag{3}$$

The product's lifetime at the time of the i-th observed failure.

$$t_i(r) = \left[t_{i-1}(r)^\beta - \alpha(r)^\beta \times LnU_i\right]^{\frac{1}{\beta}} \tag{4}$$

In the above equations, where β is the shape parameter, and Ui is a random variable following a uniform distribution in the interval [0,1].
Assuming that the repair cost for product failures is a function of the time of failure and the product's operational performance at the time of failure, the total cost of product failures is calculated as follows.

2.1. Calculation of the Repair Costs during the Base Warranty Period:

Let's assume that the repair cost for product failures depends on the product's lifetime (t) and its operational performance ($u=r\times t$) at the time of failure. In this case, the cost function for each repair under the usage rate $R=r$ is calculated using the following relationship:

$$C(t|r) = c_0 + c_1 x^\omega (rt)^\xi \tag{5}$$

where c_0, c_1, ω, and ξ are parameters of the cost function that can be estimated using historical data.
Taking into account the simulation of the two-dimensional product failure process and the prediction of the repair cost for each failure, the Monte Carlo simulation steps will be as follows:

Fig. 1. Pseudo Code for Estimating Two-Dimensional Base Warranty Service Costs

Simulation Pseudo Code

```
Set simulation number SimNum
for s=1 to SimNum
  Generate usage rate random number r
  Compute TBW/r
    Set product age t =0
  while (t<  TBW/r )
        Compute the age of product at the time of failure based on Eq.4
      if (t < TBW/r )
        Update the number of failure during BW period
        Compute the cost of failure using Eq. 5
        Update Total cost of warranty repairs
      end
    end
end
```

It seems that using Monte Carlo simulation allows for the straightforward calculation of product failure costs during the warranty period. In the following section, a numerical example will be provided to explain the simulation process along with the results.

3. NUMERICAL EXAMPLE

In this section, let's assume that a manufacturer producing a two-dimensional base warranty product, such as a car, aims to estimate the service costs during the base warranty period. Drawing from reported research in the warranty literature, suppose the random variable for the time to the first observed product failure follows a Weibull distribution, and the random variable for the customer's product usage rate follows a gamma distribution. The parameter values along with the repair cost parameters during the base warranty period are presented in the table below:

Table 1 - Model Parameter Values

Values	Specification of random variable or parameter
$W_1 = 2\ (Year)$or$U_1 = 40\ \ (10^4 Km)$	Two-dimensional basic warranty coverage
Free minimal repair	Warranty policy
$X{\sim}Weibull\ (\alpha =\ 0.8666, \beta =\ 1.6237\ )$	the first The random variable is the time until failure of the product
$X{\sim}Gamma\ (\alpha = 8.6277\ , \beta = 1.9050\ \ )$	Random variable of customer usage rate of the product
$r_D = 17\ (10^4 Km/Year)$	The customer's nominal usage rate of the product
$\gamma = 1.15$	AFT parameter
$c_0 = 10, c_1 = 40, \omega = 0.55, \xi = 0.4$	Parameters of the failure repair cost function

Taking into account the specified parameter values and implementing the simulation algorithm in the MATLAB software environment for SimNum=20000, the following results are obtained. The histograms below depict the number of product failures during the warranty period and the histogram of two-dimensional base warranty costs.

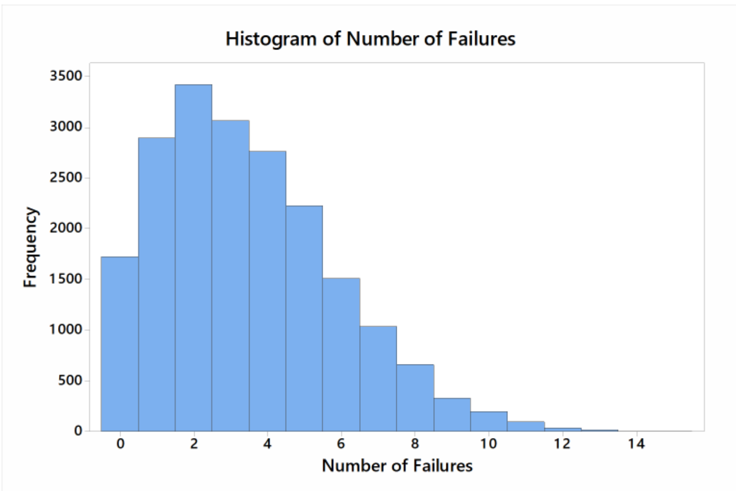

Fig. 2. Histogram of the Number of Failures during the Two-Dimensional Base Warranty Period

As observed in Figure 2, the number of potential product failures can vary from 0 to 13. The expected value of product failures with the specified parameters in Table 1 is 3.47 failures. Therefore, the manufacturer needs to be prepared to compensate and repair this number of failures for each product.

Now, the question is, what would be the cost of repairing such failures during the base warranty period? Assuming that the repair costs for these failures can be estimated from the function introduced in Equation 5, the histogram of warranty costs using Monte Carlo simulation is estimated as follows.

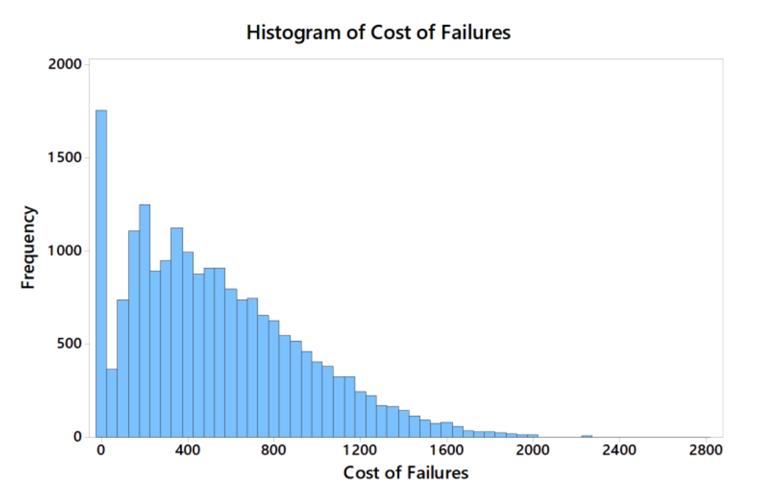

Fig. 3. Histogram of Warranty Service Costs during the Two-Dimensional Base Warranty Period

Upon examining the chart in Figure 3, it can be noted that the repair costs can span a wide range from zero to 2400 monetary units. Such a broad range indicates a high risk of exceeding the expected service cost. This highlights the necessity of paying careful attention to service cost and service management. Notably, in other approaches commonly used in the warranty literature, the range of cost variations is not as emphasized, potentially leading to financial losses for the manufacturer.

In the presented example, the average cost is estimated to be 541.60. As observed, the range of service costs is much broader than the expected value, allowing the manufacturer to observe and plan for the associated risks by examining the cost histograms. This approach provides a more comprehensive view of the risks related to service costs, allowing the manufacturer to plan and address them effectively.

In the concluding section, a summary of the discussion is presented.

2. CONCLUSIONS:

The majority of reported articles in the field of base warranty focus on mathematical modeling of the failure process and estimating the expected cost of product failures during this period. However, the limitations of mathematical modeling, combined with an exclusive emphasis on the expected cost, have resulted in neglecting the risk of exceeding the anticipated service costs.

In this article, leveraging the advantages of Monte Carlo simulation, the two-dimensional base warranty product failure process was extracted. Assuming that the repair cost for failures is a function of the product's lifetime and operational performance at the time of failure, the repair costs were estimated. The use of simulation not only simplifies cost estimation but also, by plotting histograms of failures and service costs, provides valuable insights into the potential variability of failures and repair costs. This perspective can assist the manufacturer in predicting and adequately preparing for the service requirements associated with such a volume of failures and securing the financial means to cover the related costs.

Transparency Statement

The data supporting this study are available upon reasonable request to the corresponding author, subject to ethical and confidentiality considerations.

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Declaration of Interest

The authors declare that they have no competing interests.

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