



Pricing Extended Warranty Contracts Considering Random Repair Costs and Time Value of Money

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ARTICLE INFO	ABSTRACT
<p>Article History: Received 6 August 2022 Received in revised form 25 October 2022 Accepted 21 December 2022 Available online 26 December 2022</p>	<p>Extended warranty contracts have become an increasingly important source of revenue for manufacturing companies. These contracts represent agreements between manufacturers and customers, allowing the latter to receive maintenance and repair services beyond the standard warranty period in exchange for a predetermined fee. Typically, the contract fee is collected by the manufacturer upfront, while the costs associated with repairs and replacements occur at future time intervals. This study addresses the pricing of extended warranty contracts by considering both variable repair costs and the time value of money. Initially, the product failure process is modeled, treating the cost of repairing accidental failures as a random variable, and estimating the expected costs of addressing future failures. Subsequently, these costs are adjusted to account for temporal financial considerations, enabling the calculation of the total expected cost of the extended warranty. Incorporating the manufacturer's desired profit margin, the appropriate selling price of the contract is determined. To demonstrate the practical application of the proposed methodology, a numerical example is presented, and the resulting pricing structure is analyzed. The findings provide valuable insights for manufacturers seeking to optimize warranty pricing strategies while accounting for uncertainty in repair costs and financial factors, contributing to improved revenue management and customer satisfaction.</p>
<p>Keywords: Extended Warranty, Cost Estimation, Time Value of Money, Pricing</p>	

1. INTRODUCTION

In today's competitive market, focusing solely on the production and sale of products is no longer sufficient to ensure the success of manufacturing companies. Complementary services that address customer needs and preferences can create new avenues for business growth, increase revenue, and enhance customer trust. In this context, the design and sale of extended warranty contracts represent a highly attractive opportunity for investment and active participation by manufacturers. The significance of these contracts is highlighted by the fact that the market value of extended warranties in the United States was estimated at over \$44.7 billion in 2017 [1]. Furthermore, while the profit margin for the production and sale of electronic products in the U.S. typically ranges from 5% to 10% [2], offering extended warranty contracts for these products can generate profits of 44% to 77%

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[3]. In the automotive sector, such contracts can contribute up to 100% of the profit to providers, including manufacturers, insurance companies, and third-party firms [4].

Based on these data, it is reasonable to infer that designing and offering extended warranty contracts, particularly in emerging markets, has strong potential for development and substantial financial returns. However, neglecting the scientific principles of contract design and pricing may not only reduce profit margins but also result in significant financial losses. Therefore, careful attention to estimating product failure processes, associated costs, and pricing strategies is essential for contract providers.

A review of the literature indicates that, compared to studies on standard warranties, research on extended warranties remains relatively limited, although interest has increased over the past two decades. Given that this article primarily focuses on failure estimation and pricing of extended warranty contracts, only a brief review of relevant studies is presented here. For a more comprehensive overview of extended warranties, readers are referred to [5–8].

Hartman and Laksana [9], using a dynamic programming approach, extracted a set of optimal policies for extended warranty offerings to customers and optimal pricing for providers. In this article, they demonstrated how a provider could maximize its profit by considering customer risk preferences and offering various options for extended warranties to customers. Tong et al. [10], based on the customer's usage rate of the product, designed and priced a set of extended warranty contracts for customers. In a case study, Musakwa [11] priced extended warranty contracts for trucks with a specific type of engine by examining fundamental engine failure data.

In order to achieve this, they employed a non-parametric model for product survival with interval-censored data. Jindal [12] also investigated the behavioral and experiential aspects of extended warranty purchases by customers, analyzing factors influencing the demand for such contracts, including customer risk preferences and the price of extended warranties. Lu and Zhan [13] examined the impact of product quality and its improvement on the manufacturer's profit and the price of extended warranties, demonstrating a strong correlation between product quality, the number of failures, profit, and the price of extended warranties. Khiabani and Rangan [14], considering minimal repairs of partial failures and replacement of essential failures during the base and extended warranty periods, derived the expected long-term provider profit function per unit time, taking into account the demand for extended warranties, and determined the optimal price for this contract. Zheng and colleagues [15] determined the price of extended warranties for cases where the extended warranty contract is traditionally sold at the beginning of the base warranty period, and the flexible price of the extended warranty contract, in which the customer purchases the contract during the base warranty period or at its end. Li and colleagues [16] examined the pricing of extended warranties in a supply chain. In this chain, the product is supplied to retailers by the manufacturer and is sold to customers. The extended warranty contract can be offered by either the manufacturer or the retailer, and the costs of compensating failures during the warranty period can be covered. In such conditions, a game-theoretic model was formulated to maximize the chain's profit, considering the sales process, price, and length of the extended warranty period, and optimal values were extracted.

If observed, none of the reported studies considered the effects of the time value of money and the random nature of repair costs in pricing extended warranties. However, on one hand, there can be a significant time gap between purchasing this contract and the time of paying for the repair costs. On the other hand, repair costs can vary depending on the type and component of the failed part. Therefore, considering these two factors can contribute to a more realistic pricing process. It is noteworthy that only in the study conducted by Teng [17], the time value of money was taken into account in determining the price of extended warranties. In this study, random times of failure were extracted, and the repair cost for the j th failure was considered equal to the average cost of failure at the time the j th failure occurred. However, it should be noted that when average cost is considered, the impact of the increase in costs due to inflation and the product's wear and tear on repair costs is not observed. Additionally, in this study, only for the case where the product failure rate is constant, an exact mathematical model was extracted, and for the case where the product failure rate is increasing, an approximate solution was used to calculate the time value of warranty costs.

Simultaneously, in this article, the effect of inflation in repair costs, along with the random nature of these costs, is considered alongside the total cost of product servicing. Then, by taking into account the expected profit margin of the manufacturer, the discount rate is calculated, and the selling price of this contract at the beginning of the extended warranty period is determined. For this purpose, first, the product failure process is extracted in the form of a non-homogeneous Poisson process. Then, by adding the random repair cost, the expected cost of servicing is extracted, and the contract price is determined.

The following sections of the article are presented as follows. In the next section, the mathematical model of product failure and estimation of product failure costs are extracted. Then, in the third section, a numerical example is presented, and the pricing process for this example is explained. Finally, the advantages of the reviewed model are presented in the conclusion section.

2. MATHEMATICAL MODEL FOR EXTENDED WARRANTY

Consider a product with a base warranty period, denoted by W_B , and assume that at the end of this period, the extended warranty contract with a duration of W_E is immediately purchased. For this product, all product failures during the base and extended warranty periods are corrected by minimal repairs, and the warranty policy for free repair of product failures is determined by the manufacturer. Let X be a random variable representing the time until the first product failure, with $f_X(x)$ and $h_X(x)$ being the probability density function and failure rate, respectively. Assuming that all product failures during the base and extended warranty periods are compensated by minimal repairs, the number of product failures during the warranty period will follow an inhomogeneous Poisson process, where the intensity function $\lambda(x)$ is equal to the failure rate function $h_X(x)$ [18]. Therefore, the expected number of failures during the extended warranty period is calculated as follows:

$$E[N_f(W_B, W_B + W_E)] = \int_{W_B}^{W_B+W_E} h_X(x) dx \tag{1}$$

With each occurrence of a product failure within the time interval $[W_B, W_B+W_E]$, the extended warranty provider also performs minimal repairs according to the warranty policy. Due to the random nature of failures, it can be expected that within a specific time period, such as the base year, the repair cost will be proportional to the type, severity, and randomly affected component of the failure. Let Y be the random variable representing the cost of repairing a failure at the moment of selling a new product (base year or year zero), and its value can be a random number in the range $[c_0^{min}, c_0^{max}]$. In this case, Y can be expressed as follows:

$$Y = c_0^{min} + (c_0^{max} - c_0^{min})B \tag{2}$$

It was mentioned that c_0^{min} and c_0^{max} are the minimum and maximum values of the repair cost at the beginning of the base warranty or at the moment of selling a new product. Let B be a random variable in the interval $[0,1]$ [0,1]. Suppose the random variable B follows a beta distribution, denoted as $B \sim \text{Beta}(\alpha, \beta)$, where α and β are the shape parameters of the distribution. In this case, the probability density function of the distribution for B and subsequently Y will be as follows:

$$f_B(b) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} b^{\alpha-1}(1 - b)^{\beta-1} \quad 0 \leq b \leq 1 \tag{3}$$

$$f_Y(y) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)(c_0^{max} - c_0^{min})} \left(\frac{y - c_0^{min}}{c_0^{max} - c_0^{min}} \right)^{\alpha-1} \left(1 - \left(\frac{y - c_0^{min}}{c_0^{max} - c_0^{min}} \right) \right)^{\beta-1} \tag{4}$$

In explaining the choice of the beta distribution as the random distribution in the interval $[0,1]$ [0,1], it should be noted that due to the flexibility of the beta distribution in this range, it can reproduce a wide range of behaviors within this interval. However, other distributions can also be used to describe potential repair costs, and with minor modifications, the model can be rewritten and adjusted for their application.

Regarding the explanation of how to derive equation (4), considering that, according to equation (2), the random variable Y is a linear function of B with a beta distribution, the cumulative distribution function (CDF) of Y can be extracted using the beta distribution. By differentiating the CDF, the probability density function (PDF) of the repair cost can be estimated. Taking into account the random cost of failures during the extended warranty period, the expected cost of failures will be as follows:

$$E[C_R(W_B, W_B + W_E)] = \int_{c_0^{min}}^{c_0^{max}} \int_{W_B}^{W_B+W_E} y h_X(x) f_Y(y) dx dy \tag{5}$$

In addition to the random nature of costs within a specific time period, the repair cost may increase over time due to rising expenses for spare parts, human resources, overhead costs, etc. Let i_f be the inflation rate of product failure costs in each time unit (such as a year). In that case, the repair cost of the product at time x is given by:

$$Y_x = Y(1 + i_f)^x \tag{6}$$

Where Y is the repair cost of the product in the base year (year zero), which is random, as mentioned earlier. By incorporating the inflation effect into equation (5), the expected cost of failures during the extended warranty period will be:

$$E[C_{RI}(W_B, W_B + W_E)] = \int_{c_0^{min}}^{c_0^{max}} \int_{W_B}^{W_B+W_E} y(1 + i_f)^x h_X(x) f_Y(y) dx dy \tag{7}$$

According to equation (7), the total cost of servicing the extended warranty can be calculated. However, it should be noted that for pricing the extended warranty contract, the contract price at the time of selling the extended warranty contract (i.e., $t=W_B$) needs to be estimated. For this purpose, the discount rate for costs must be adjusted based on the selling time. Assuming the discount rate is i , the price of the extended warranty contract can be calculated from the following relationship.

$$E[Price] = \int_{c_0^{min}}^{c_0^{max}} \int_{W_B}^{W_B+W_E} \frac{y(1 + i_f)^x}{(1 + i)^{x-W_B}} h_X(x) f_Y(y) dx dy \tag{8}$$

To determine the discount rate, assume that the provider wishes to earn a profit margin equivalent to r percent based on the value of money at the time of selling the contract. In this case, the discount rate must be calculated in such a way that, for that rate, the selling price at time $t=W_B$ with a profit margin of r percent is estimated. For this purpose, the discount rate must be calculated from the equality of the income at the time of sale with a profit margin of r percent and the adjusted cost with the discount rate i . This equation will be as follows:

$$(1 + i_f)^{W_B} (1 + r) = \frac{(1 + i_f)^x}{(1 + i)^{x-W_B}} \tag{9}$$

By solving the above equation, the discount rate is calculated as follows:

$$i = (1 + i_f)(1 + r)^{\frac{-1}{x-W_B}} - 1 \tag{10}$$

Therefore, by considering the repair cost as a random variable and also incorporating the inflation effect of costs, the price of the extended warranty can be estimated at the time of its sale, which is the end of the base warranty and the start of the extended warranty contract, from equation (8). In the next section, a numerical example is presented to illustrate the model

3. NUMERICAL EXAMPLE

For a specific product under the base warranty, suppose $W_B=2$ years is the length of the base warranty, and the manufacturer intends to offer an extended warranty contract with a length of $W_E=1$ year at the end of the base warranty. According to this contract, product failures during the extended warranty period are compensated with minimal free repairs. Previous experiences and field data analysis have shown that the time to the first product failure follows a Weibull distribution $X \sim \text{Weibull}(a=1.12, b=1.806)$ (in years), and the repair costs are random based on the base year (the year the new product is sold to the customer) and follow a function of the Beta distribution $B \sim \text{Beta}(\alpha=2, \beta=3)$ in the interval $[c_0^{\min}=50, c_0^{\max}=500]$ (in monetary units).

Assume that the annual inflation rate for service costs $i_f=15\%$, and the manufacturer aims to achieve a profit margin of $r=25\%$ from the sale of the extended warranty contract. In this case, the price of this contract is determined by the following relation:

$$E[C_R(W_B, W_B + W_E)] = \int_{c_0^{\min}}^{c_0^{\max}} \int_{W_B}^{W_B+W_E} \frac{y(1+i_f)^x}{(1+i)^{x-W_B}} b a^{-b} x^{b-1} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)(c_0^{\max} - c_0^{\min})} \left(\frac{y - c_0^{\min}}{c_0^{\max} - c_0^{\min}}\right)^{\alpha-1} \left(1 - \left(\frac{y - c_0^{\min}}{c_0^{\max} - c_0^{\min}}\right)\right)^{\beta-1} dx dy \tag{11}$$

By solving the above equation, the value of the extended warranty price is calculated. In the following table, the expected number of failures during the extended warranty period, the expected cost without inflation, the inflation-adjusted cost at the end of the extended warranty period, along with the price of the extended warranty at the time of purchase, are presented.

Table 1. Model Parameter Values

Estimated Parameter	Parameter Value
Expected Failures During Extended Warranty	3.0770
Expected Inflation-Adjusted Service Cost	1080
Expected Discounted Service Cost	936
Extended Warranty Price	1170

It can be expected that the price of the extended warranty contract depends on both the base and extended warranty periods. Therefore, the impact of changes in each of the above parameters on the contract price will be examined and analyzed in the following. The results of these variations are presented in Figures 1 and 2.

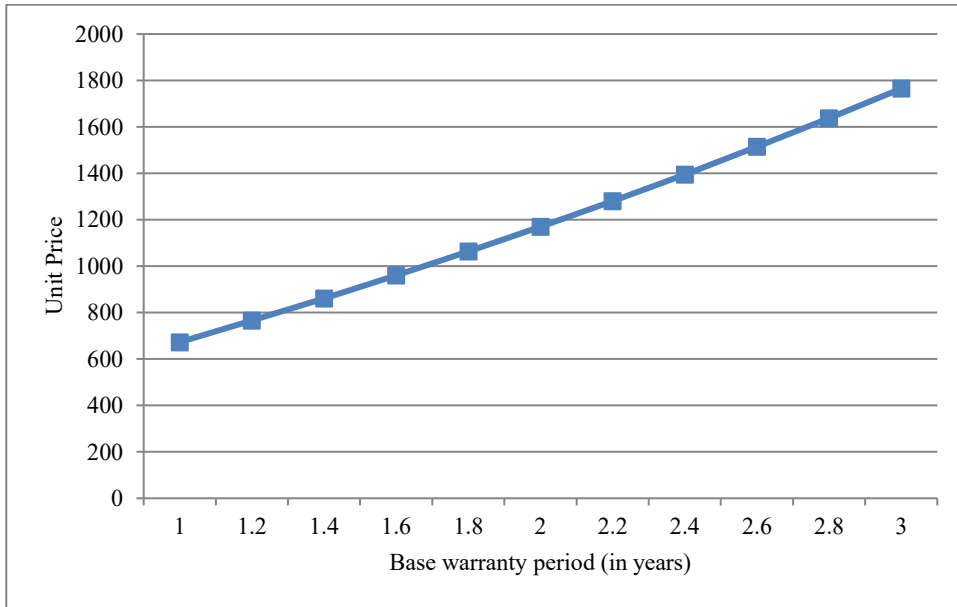


Fig. 1. The Effect of Increasing the Base Warranty Period on the Price of Extended Warranty for $W_E=1$ (year)

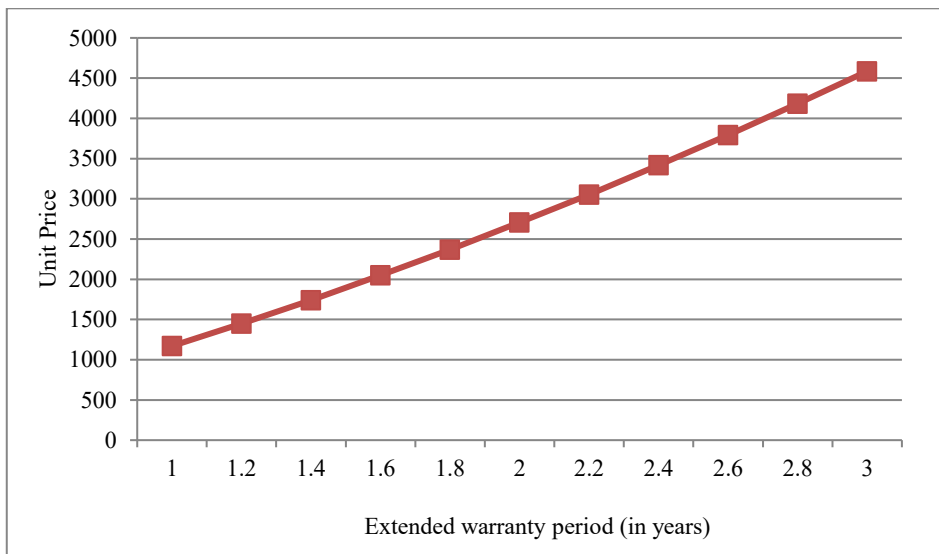


Fig. 2. The Effect of Increasing the Extended Warranty Period on the Price of Extended Warranty for $W_B=2$ (years)

As observed in Figures (1) and (2), it can be expected that with an increase in the base and extended warranty periods, the price of the extended warranty will increase. To explain the reason for the increase in the price of the extended warranty with an increase in the base warranty period, assuming the extended warranty period is constant, it should be noted that lengthening the base warranty interval causes the product to have a higher lifespan during the extended warranty coverage period and, consequently, a higher failure rate. However, by comparing the two figures, it can be inferred that, assuming linearity in both curves, the slope of the curve in Figure (2) is greater than that in Figure (1), and naturally, an increase in the extended warranty period by one unit of time results in a greater increase in the contract price compared to an increase of one unit in the base warranty. In the following section, the conclusion will be presented.

4. CONCLUSION

The sale of extended warranty contracts by the manufacturer can provide an excellent opportunity for better utilization of the after-sales service network and profit from providing these services. However, a thoughtful consideration in regards to these contracts is their proper pricing to encourage customers to purchase them while ensuring reasonable profits for the manufacturer. In this article, for a hypothetical product under extended warranty, the product failure process was extracted, and assuming the random nature of repair costs, the expected cost of the product during the extended warranty period was estimated. Then, by considering the time value of money and adding the effect of inflation on service costs to the model, the expected service costs, adjusted for inflation, at the end of the contract were estimated. Subsequently, by calculating the discount rate for the desired profit margin of the manufacturer, the price of the extended warranty at the beginning of this period was determined. With the presentation of a numerical example, the results of the examined model and the impact of warranty periods on the pricing of extended warranty contracts were studied.

Transparency Statement

The data supporting this study are available upon reasonable request to the corresponding author, subject to ethical and confidentiality considerations.

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Declaration of Interest

The authors declare that they have no competing interests.

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