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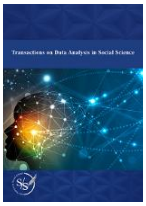

A Predictive Model for Optimizing Claims and Risks in the Tendering Stage of Construction Projects

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ABSTRACT

Participation in tenders and investment in construction projects involves numerous factors and significant complexities. Decision-making in this context is highly sensitive and critical due to constant fluctuations in the economic conditions of the construction industry and the target investment environment. Consequently, companies require accurate information and rigorous data analysis, often expending substantial time and financial resources on human expertise to address these challenges. It is evident that a comprehensive and precise evaluation of project conditions prior to winning a tender plays a fundamental role in achieving ultimate success for all project stakeholders. To address this need, and within the context of Iran's construction industry, key decision-making variables were identified to develop a deterministic mathematical model based on an operational research approach. The model incorporates four primary variables: 1) maximum investment cost and bid price, 2) project duration and associated time-based costs, 3) projected net profit, and 4) likelihood of claims occurrence and related claim costs. Using this model, tender decisions can be assessed according to overall price indices, construction duration, expected profit, and potential claims with their corresponding costs. Ultimately, it is concluded that such a model cannot rely solely on analytical operational research solutions and must be calculated using numerical approaches and approximations.

1. INTRODUCTION

Identifying the factors that influence participation in construction project tenders is often ambiguous and unclear, and it is evident that decision-making regarding investments in this area is highly critical and sensitive. To address

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this, one may look to experiences documented by others in the scientific literature. These challenges are recognized worldwide, although the body of research addressing construction-related tender issues remains relatively small, despite the widespread nature of decision-making difficulties in tenders on a global scale. Internationally, this topic has been discussed for several decades, and scientific efforts to support and improve tender-related decision-making are ongoing.

In this regard, Paeck and Ock (1993) stated that "construction projects, such as environmental restoration, public transportation system projects, and occasionally tunnel construction projects, are complex and risky; contractors face situations involving numerous unknown, unexpected, often unfavorable, and frequently unpredictable factors [1]. This means that the level of ambiguity is so high that it can overshadow the entire decision-making process, potentially resulting in costly decisions. The increased costs can be attributed to uncertainty in various risk analysis elements, such as limited engineering data, insufficient construction knowledge, and lack of documentation on operational history."

A critical issue associated with this high level of complexity, which has persisted for decades, is the "sheer number of highly interrelated variables required to reach a decision" [2]. The importance of this issue can be understood through the cascading consequences of such decisions: "…a decision made regarding any project has a significant impact on the company's short-term profit, which in turn affects the company's long-term strategy and performance." Therefore, this topic demands careful examination: "…tender decisions provide a link between strategy and individual project plans… [and] factors related to the subject matter include the lack of clear alignment between the project and the organization's key strategic priorities" [3].

From this discussion, it can be inferred that the lack of a robust, objective solution for tender-related decision-making is generally due to the following reasons [4]:

- Limited research literature and empirical studies.
- Context-specific investment targets influenced by geographic or political factors.
- Absence of comprehensive historical databases.
- Pre-tender cost estimations.
- Reliance on subjective intuition.

Accordingly, the scarcity of research literature and the limited availability of relevant data—ideally collected from past tenders—highlight the need for deterministic functions and mathematical models capable of providing a clearer representation of a potentially successful bid.

Therefore, this study aims to develop a mathematical function to create a deterministic model for an operational research (OR) approach to assess whether, and under what conditions, investments in construction projects should be proposed. This model is based on objectively measurable contract variables and seeks to optimize the likelihood of tender success and project profitability.

2. RELATED WORK

Risk and claims management during the tendering phase of construction projects is one of the key challenges in the construction industry, as it can significantly impact project cost, schedule, and quality [5]. Studies indicate that contractual, financial, and operational risks during this phase may lead to increased claims and project delays [6]. For instance, predictive models based on artificial intelligence, such as artificial neural networks (ANN), have been proposed for analyzing contractual risks, enabling automated risk assessment [7].

In the field of claims management, research has identified factors such as inaccurate material estimation, financial issues, and project time constraints as major contributors to cost escalation [8]. Project performance prediction models based on dynamic systems can anticipate the causes of claims and improve overall project outcomes [9]. Additionally, machine learning algorithms have been applied to analyze the causes of claims in large-scale highway projects, enhancing predictability and control over disputes [10].

Construction supply chain risks have also attracted attention, where traditional methods such as surveys and case studies are combined with AI approaches to identify, assess, and prioritize risks [11]. Contractual risk mitigation models in infrastructure projects explore significant relationships among risk factors using structural equation modeling [12]. Predictive analytics in construction, leveraging data mining and machine learning algorithms, can forecast risk trends [13]. Claims management systems based on EPC principles and optimization algorithms aim to optimize multiple objectives, including reducing project costs and duration [14].

Neural network-based predictive models for claims can estimate the potential impact of disputes, thereby enhancing decision-making awareness [15]. Risk assessment for cost escalation in construction projects using machine learning algorithms addresses domain-specific challenges [16]. Claims reduction programs that emphasize organizational risk culture have been shown to improve customer satisfaction and increase profitability [17]. Cash flow optimization models can predict environmental risk factors affecting project management [18]. Predictive modeling of cost escalation using support vector machines relies on archival project data [19]. Finally, comprehensive reviews of construction risk management research identify the intellectual structure of the field and emerging themes, providing a roadmap for future studies [20].

2.1. Familiarity with the Deterministic Modelling Approach

In this study, a constructive deterministic approach was employed to evaluate decision-making variables with greater precision. With this type of modeling, “decision-makers are increasingly interested in understanding the uncertainty of models,” which is a critical feature of determinism in mathematics and science in general [21].

In essence, determinism “is an approach that views causality as a complex and continuous process of interaction among abstract forces, which manifest themselves in specific material ways within particular historical and geographical contexts, producing outcomes that, in turn, feed back into the causality process itself” [22]. It should be noted that determinism refers to an abstraction inherent to a model, rather than a fixed property of any particular physical realization of a system [1].

Regarding deterministic mathematical models, for analytical clarity, the time series of events or derivation of variables is referred to as the “state” of the system. Then, in any system with a deterministic nature, “measurements are performed on this system, which may represent the states themselves or combinations thereof” [23].

Formally, the general formulation of a deterministic model can be expressed as follows:

A “general, unstable boundary function for any quantity u” can be defined by the following differential equation within a domain Ω, subject to boundary conditions specified on the boundary Π.

$$\text{Ł}\ (x, y, z, t) = f(x , y ,z, t) \text{ for } (x, y, z, t) \in \Omega \tag{1}$$

$$\text{Ł}_u(x, y, z, t) = g(x, y, z, t) \text{ for } (x, y, z, t) \in \prod \tag{2}$$

where Ł and Łu are defined as differential operators, and f and g are prescribed functions. The initial condition must also be incorporated into these equations [24].

3. METHODOLOGY

As previously noted, the objective of this study is to introduce a mathematical function capable of deterministic modeling to optimize and provide the most accurate estimation of the validity of bidding decisions. To achieve this goal, it is first necessary to identify the relevant variables. These are the variables that “meet demand at minimum cost or determine which investments are feasible under existing conditions.” This indicates the need for a mathematical and operations research (OR) model to support optimal decision-making [25].

In this context, the study applies the following concepts:

- **Mathematical modeling:** "A set of variables and relationships necessary to describe the essential characteristics of a specific problem."
- **Analytical study of complex systems:** "The examination of how mathematical models are formed from complex scientific, engineering, industrial, and management problems, and how these models are analyzed using mathematical techniques."

In short, as a methodological fusion, this research applies deterministic operations research, where all parameters are fixed [21]. Consequently, the final model and resulting decisions are not random. Using OR modeling, we aim to construct a function represented by decision variables.

The following steps outline the methodological approach to develop the mathematical function:

- Problem definition and data collection
- Mathematical model formulation
- Solution of the mathematical model
- Application of the model for real-world prediction

From this point onward, each step is implemented as described.

3.1. Problem Definition and Data Collection

Given the core problem—constructing a deterministic decision-making model for bidding—decision-related variables regarding whether to submit a bid must be identified. Accordingly, the criteria outlined in the introduction are operationalized into the following variables, enabling objective quantitative analysis within the decision-making process.

The data for these criteria were collected via surveys conducted with contractors operating in Iran, validated by the Office of Strategic Planning and Control at the Presidency, and then logically mapped to the variables of the model.

3.1.1. Investor Status – Maximum Investment and Bid Price

Each company typically determines a financial bid for proposed investments based on analysis of available or historical data across different time series. This bid serves as the potential investment baseline for the company. It is informed by the company's technical capacity and historical success rates, which ultimately impact turnover and profit margins. These represent the capital resources convertible into actual investment amounts.

3.1.2. National Currency and Inflation Rate – Project Duration and Time-related Costs

Given the volatile nature of variables such as inflation and currency balance in Iran, these factors can, for modeling purposes, be represented as time variables. Despite significant fluctuations in monetary values, time can be converted to real financial cost using appropriate coefficients. Therefore, instead of directly using monetary variables, time is considered in the model.

3.1.3. Profit Margin – Expected Net Profit

The quantitative calculation of net profit represents the ultimate objective for each contracting entity. These indices must be as accurate as possible to avoid underestimating the expected profit.

3.1.4. Risk Management and Contractual Liabilities – Claim Probabilities and Associated Costs

Since the project owner often delegates the project to primary contractors, who may in turn subcontract portions to secondary contractors, multiple layers of risk are introduced. The cumulative approach to managing claims, according to contract protocols, allows for risk estimation prior to potential litigation. Considering the substantial costs associated with legal action, anticipating claim probabilities and costs is central to risk management in this study.

As previously noted, to construct a function capable of accurately estimating bidding decisions relative to success probability, a deterministic mathematical model must be developed. The aforementioned criteria are translated into numerical variables, which are then mapped in the model. Given the non-linear nature of these variables, the final approach employs linearization of specific non-linear models, incorporating absolute values and min/max objective functions [26]. Ultimately, the fully defined mappings are examined within the model to validate performance, making it suitable for deterministic OR applications.

3.2. Mathematical Modeling

The final bid price, along with other influential qualitative criteria introduced here, serves as an input for the ultimate optimization model [27].

3.2.1. Price Element Function

From an investment logic perspective, each bid for civil engineering projects represents an absolute cost for the project owner and initial expenditure for contractors and supervisors, considering the required cash flows. Therefore, the relationship between the estimated price range and the corresponding variable in the final model must be relative, reflecting both the probability of success and the comparative scope.

Since the goal is to construct a relative evaluation function, a multi-rule function is applied as follows:

$$\Pr(p) = \begin{cases} 1)\ 0\% \ < p < 20\%\ of\ total\ annual\ revenue/profit \\ 2)\ 20\% < p < 40\%\ of\ total\ annual\ revenue/profit \\ 3)\ 40\% < p < 60\%\ of\ total\ annual\ revenue/profit \\ 4)\ 60\% < p < 80\%\ of\ total\ annual\ revenue/profit \\ 5)\ 80\% < p < 100\%\ of\ total\ annual\ revenue/profit \end{cases} \tag{3}$$

Where p represents the bid price and Pr denotes the system's reasonable price under a given tender scenario. In other words, if the final price (calculated using the detailed estimation methods and concepts described) constitutes a significant portion of the company's annual profit margin and/or revenue (based on each company's financial statement), it is considered a high bid (high risk). Conversely, if it represents a minor share, it is regarded as a low bid (low risk).

Since the output of this variable should reflect the qualitative value of such an assessment—indicating whether the proposed bid is good/appropriate or bad/inappropriate—normalizing the value between 0 and 1 simplifies the modeling process. Accordingly, the price variable for the final optimization model is introduced through the following function:

$$P(Pr) = \frac{Pr}{Pr^{Pr}} = \frac{1}{Pr^{Pr-1}} \tag{4}$$

The mapping of the first variable, without aggregating it to a total and while maintaining the same explanatory logic, receives discrete values ranging from 1 to 1/625, reflecting real-world tender conditions.

3.2.2. Project Duration

One of the most critical aspects in construction projects is time planning and translating time into cost within a capital-driven approach. In general, project duration is considered a costly factor for all stakeholders involved in a contract; thus, any reduction in it can be significant. In practice, any extension in project time corresponds to an increase in project costs. Therefore, the primary calculations must reflect an inverse relationship between project duration and the final decision made by the OR model.

To incorporate a meaningful and normalized element of interest rate, we define a ratio between the target profit margin and the official bank interest rate or traditional discount rate for informal investments, integrating it into the model function. Since small fractional changes in time variables can be highly significant due to the typically large duration ranges, we employ non-linear exponential functions. Accordingly, the time variable function is introduced based on the annual profit value and scaled to seasonal variations as follows:

$$\tau(t) = \frac{1}{4}\left(\left[\frac{t}{3} + 1\right]\right),\ T(\tau(t)) = \left(\frac{1}{\tau(t)^{e}}\right) * \left(\frac{PM}{BR}\right) * RP \tag{5}$$

T: Estimated project duration
PM: Target profit margin
BR: Annual official bank interest rate
PR: Total contract value

In the project duration evaluator, the inputs are entered as their monthly equity values within an annual calendar. Specifically, a constant of 1/4 is used to preserve seasonal proportions, while another constant of 1/3 is applied to maintain monthly granularity within each quarter. For example, if the project duration is estimated between 0 and 3 months, the input value will be 1/4. Similarly, if the estimate is between 6 and 9 months, the input value will be 3/4, and so on. However, prior knowledge still applies regarding the 1:1 characteristic of such value distributions within each range class, ensuring consistency across interval categories.

3.2.3. Project Net Profit

Although the projected profit is usually considered straightforward, misestimations in operational costs and calculation errors can seriously threaten contractors' expectations from the transaction [28]. To address such issues, the standard approach is to incorporate a fixed overall profit margin into the final bid after all cost estimations. Accordingly, the net profit calculator in the optimization framework is defined as follows.

As previously noted, any error in accurately accounting for potential risk factors can significantly reduce profit. Therefore, the target profit margin must account for the number of subcontracted tasks or subprojects within the overall process. This leads to the following function:

$$B(X(n)) = (PM)^{X(n)} \tag{6}$$

Where:

PM: Desired profit margin
X(n)*:* Individual execution units

3.2.4. Claim Probability and Loss

In the approach presented in this article, claims constitute the most significant and evidently costly risks in construction project contracts. Evaluating and quantifying these variables—including the probability of occurrence and the associated cost for each claim—are key components of the optimization function. Clearly, there is a dynamic relationship between these two variables, and their output, in conjunction with the appropriate level of investment, directly affects decision-making.

Although these two variables are calculated nonlinearly, they are integrated as a single variable into the final OR model. Logically, claims with a low probability and low cost are in a more favorable position than claims with high probability and high cost. Other combinations are distributed across the same range according to the same logic.

This results in a nonlinear exponential relationship between the indicators within the final OR modeling. The exponential component reflects the stress impact of these variables according to our approach.

Moreover, for integration with the overall framework, claim costs are treated as a qualitative variable based on historical data, divided into five distinct levels: one representing the lowest cost and five representing the highest-cost claims. This approach is expressed in the function as follows:

$$Ex(CL(X)) = \frac{1}{[Ex\ (cl)]^{(CL(X))}} \tag{7}$$

EX: Cost of each claim
CL(X): Probability of occurrence of each claim

3.3. Mapping and Mathematical Model Rule

It is now time to take a closer look at the mathematical mapping used to construct the deterministic model. Based on the theory of functions in real number spaces of arbitrary dimension, there exists a continuous mapping between a given space and any interval of real numbers. In other words, multiple inputs can be mapped to single values within a closed interval [29].

In the context of the auction problem addressed in this study, this mathematical fact implies that any multivariable function—where its variables are the mathematical mappings introduced to compute and evaluate parameters relevant to the auction optimization process—can be normalized to the interval [0,1]. This property ensures that the auction evaluation process is meaningful in relation to the mapping of the function into probability structures. It also facilitates statistical testing for assessing optimization responses.

As previously mentioned, the function takes five independent input variables. These five variables, in brief, are:

- **Bid Price Function** P(X)
- **Execution Time** T(X)
- **Net Profit** B(X)
- **Claim Probability** CL(X)
- **Claim Costs** EX(CL)

Therefore, the deterministic mapping function O() is defined as follows:

$$O(P, T, B, CL, Ex) \Rightarrow (0, 1]; \tag{8}$$

This mapping is defined from the matrix space R5 to the interval (0,1] within R (the real number space). We now move toward solving the model for optimization. However, prior to that, it is necessary to examine certain algebraic criteria to establish the mathematical foundation of the proposed function.

3.4. Information on mapping parameters

In seeking the final output of the optimization model normalized over the interval [0,1], one can employ one of three approaches for shifting and/or mapping in continuous, one-to-one functions:

1. Linear method with integer coefficients,
2. Nonlinear method with degree 2 or higher,
3. Exponential expressions.

Considering the real conditions of the problem, final proposals are often decided through very close calls and narrow differences. Therefore, magnifying numerical values is an appropriate approach. Given the differential rate of exponential functions, the third method is employed in this model.

Due to the focus of this study on weighting mid-range claims as the most expensive and on unexpected costs, the overall model range, which maps to [0,1), is divided into two halves. The first half—[0,0.5)—is allocated to the first three variables (i.e., price, time, and profit margin), while the second half—[0.5,1)—is assigned to claim probabilities and claim costs. Consequently, based on the suggested standard deviation weighting, these last two variables are given the highest weight relative to the first three variables.

Thus, based on the information and derivations presented so far, the model can be represented as follows: Assume

$$P\big(pr(p)\big) = \frac{1}{pr(p)^{(pr(p)-1)}}$$

$$T\big(\tau(t)\big) = \frac{PM \times RP}{[\tau(t)^{e} \; * \; PR]} \quad Where\ e\ is\ the\ Euler's\ number, \qquad \tau(t) = \frac{1}{4}\left(\left[\frac{t}{3} + 1\right]\right) and\ for$$

$$t < 60{:}\;\; T(\tau(t)) \; < \; 4^{e}\,\left(\frac{PM \times RP}{BR}\right)$$

$$B(X(n)) \; = \; (PM)^{X(n)}$$

$$H \; = \frac{1}{Ex(cl)^{CL(X)}}$$

(9)

Then we have:

$$F \; = \frac{1}{11} * B \; + \frac{3}{22} * \log(T)_{K} \; + \frac{3}{11} * P, \qquad where\ K \; = \; 4^{e}\left(\frac{PM \times RP}{BR}\right)$$

$$S \; = \log 5_{2}$$

$$G \; = \; H^{\frac{1}{S}}$$

Valuation function:

$$\text{(VF)} = \text{F} + ½\text{*G} \tag{10}$$

3.5. Function Examination: Consistent and One-to-One Definitions

It is now necessary to examine whether the final deterministic model qualifies as a *consistent function*. This mathematical property ensures that the model operates correctly as a functional mapping. Subsequently, the *one-to-one (1–1) property* is assessed, which guarantees that the function assigns exactly one unique input to a distinguished point in the output domain. This property renders the model meaningful for decision-making, since no two distinct conditions will yield the same response.

For each mapping of variables, the aforementioned properties must be verified before establishing the model's final consistency. To achieve this objective, the following definitions are introduced:

Definition 1

f(x) is consistent *if and only if*:

$$\forall x,y \in D(f), \text{if } x=y \Rightarrow f(x)=f(y) \tag{11}$$

Definition 2

f(x) is one-to-one (1–1) *if and only if*:

$$\forall x,y \in D(f), \text{if } f(x)=f(y) \Rightarrow x=y \tag{12}$$

Proposition A. Every function of the form y=f(x) is well-defined. In other words, as long as a variable appears only once in a mapping, it is guaranteed that the mapping represents a function, and each variable corresponds to exactly one value in the domain.

Every function of the form y=f(x) is thus well-defined. That is, when a mapping contains a unique variable, it is ensured that the mapping constitutes a function, and each variable returns only a single value in the domain.

Suppose f and g are two well-defined functions. In this case, their compositions (fog and gof) are also well-defined.

Proof:

$$\forall x, y, if\ x = y \Longrightarrow g(x) = g(y)\ because\ g\ is\ a\ function \Longrightarrow f\big(g(x)\big) = f\big(g(y)\big)\ because\ f\ is\ a\ function \Longrightarrow fog\ is\ a\ well-defined\ function \tag{13}$$

Therefore, for any mapping involving a variable, the above criteria must be verified.

Now, consider P(X):

It consists of only one variable, and hence, according to Proposition A, it is a well-defined function.

$$pr(p) = \begin{cases} 1 & if\ 0 \le p < 20\% \\ 2 & if\ 20 \le p < 40\% \\ 3 & if\ 40 \le p < 60\% \\ 4 & if\ 60 \le p < 80\% \\ 5 & if\ \quad p \ge 80\% \end{cases} P(pr(p)) = \frac{1}{pr(p)^{pr(p)-1}} \quad \Longrightarrow \quad P\big(pr(p)\big) = \begin{cases} 1 & if\ 0 \le p < 20 \\ \frac{1}{2} & if\ 20 \le p < 40 \\ \frac{1}{9} & if\ 40 \le p < 60 \\ \frac{1}{64} & if\ 60 \le p < 80 \\ \frac{1}{625} & if\ \quad p \ge 80 \end{cases} \tag{14}$$

It may initially seem that the above function is not one-to-one (since, for example, one could argue that for the inputs p=10% and p=15%, the function returns the same value). However, it must be noted that, according to the definition of the function, these intervals within the domain do not diminish the one-to-one property of the codomain. This is due to the very purpose for which the function has been defined. In this regard, all points within a given interval of the domain are assigned the same output value. Consequently, no distinction is made between any values in the range of 0% to 20% in the first interval, as they are all considered identical inputs from the perspective of the function. The same reasoning applies to all other intervals of the domain. Nevertheless, we now proceed to examine the one-to-one property in detail.

T(x) consists of only one variable; therefore, according to Proposition A, it is a well-defined function. This variable, which represents time, is transformed—due to practical considerations—from its naturally continuous form into a discretized version. It is reasonable to clarify this practical advantage and to demonstrate that such a transformation does not reduce the integrity of the problem. It is well known that exponential functions exhibit extremely high growth rates; in other words, for large input values, these functions return disproportionately large outputs. This, in turn, creates difficulties when handling large numbers and imposes excessive computational loads on each processing step.

In the case of T(x), if the domain of this function were to accept the number of days as its scaled variable, the resulting input values for construction projects would be unreasonably large, leading to impractical computational demands. As a remedy, the scale of this variable must be reduced to ensure feasible input values. For this reason, as previously noted, the calendar year is divided into four quarters, and this quarterly division is employed as the metric scale for the input of the function.

$$\tau\,(t) = \begin{cases} \frac{1}{4} & if\ 0 \le t < 3 \\ \frac{1}{2} & if\ 3 \le t < 6 \\ \frac{3}{4} & if\ 6 \le t < 9 \\ \dots & \dots \end{cases} \tag{15}$$

$$\Rightarrow \tau(t) = \frac{1}{4}\left(\left[\frac{t}{3}+1\right]\right)\ where\ [t]\ is\ floor\ function \quad function\ \text{II}: T\big(\tau(t)\big) = \frac{1}{\tau(t)^e} \times \frac{PM \times RP}{BR}$$

$$\Rightarrow \forall\, \tau, \pi\ if\ T(\tau) = T(\pi) \Rightarrow \frac{1}{\tau(t)^e} \times \frac{PM \times RP}{BR} = \frac{1}{\pi(t)^e} \times \frac{PM \times RP}{BR}$$

$$\Rightarrow \frac{1}{\tau(t)^e} = \frac{1}{\pi(t)^e} \Rightarrow \tau(t)^e = \pi(t)^e \Rightarrow \tau(t) = \pi(t) \Rightarrow$$

$$function\ \text{III}: \mathrm{B}(X(n)) = (PM)^{X(n)}$$

The function B(x) consists of only a single variable; therefore, according to Theorem A, it is a well-defined function.

$$\forall x, y\ if\ B(X) = \mathrm{B}(Y) \Rightarrow (PM)^X = (PM)^Y \Rightarrow Ln((PM)^X) = Ln((PM)^Y) \Rightarrow X.Ln(PM) = Y.Ln(PM) \Rightarrow X = Y \Rightarrow 1-1 \tag{16}$$

$CL\ (X)$ & $Ex(CL)$:

Ex(*CL)* is a composite function composed of two single-variable functions and, therefore, according to Prop b, it is a well-defined function.

$$function\ \text{IV}: ex(cl) = \begin{cases} 1 & cheapest\ claim \\ 2 & \\ 3 & \\ 4 & \\ 5 & most\ expencive\ claim \end{cases} \quad ,\ function\ \text{V}: Cl(X),\ 0 \le Cl \le 1 \tag{17}$$

It should be noted that the fourth function is a single-point constant and the fifth function represents a probability; therefore, both of these functions are one-to-one. The function used in the model, which links these two functions, is expressed as:

$$Ex\big(ex(cl), Cl(X)\big) = \frac{1}{ex(cl)^{Cl(X)}} \tag{18}$$

Since ex is a constant five-valued function, the main function Ex simplifies to $\frac{1}{a^{Cl(X)}}$, where a takes values from 1 to 5. Consequently, its behavior is analogous to the third exponential function, and it is therefore one-to-one.

3.6. Well-Defined Ness of the Function

Remark 1: It can be shown that any function of the form y=f(x) is well-defined [1]. In fact, when a relation involves only a single variable, it is guaranteed to be a function, producing exactly one output for each input.

Now, considering all the variable functions together and returning to the final model VF:

As previously explained, the function VF is composed of two subsets, F and G. The subset F itself is a linear combination of the first three variable functions (P, T, B) with respective coefficients 1/11, 3/22, and 3/11, plus a logarithmic coefficient cap Ts to control the growth rate and keep the total value of F within the desired interval [0,0.5], taking into account the intervention weight assigned to claim-related factors in the main problem.

Regarding the function G, since EX attains a maximum value of 1 and a minimum of 1/5, it must be mapped into the desired interval (0,1) such that its minimum corresponds to 0.5. Using the exponential function toolkit, G is defined as a base raised to the power S, ensuring that the minimum value of G in the domain is 0.5. Consequently, it preserves the intended properties and remains one-to-one.

The VF function, as a projector of F and G onto the interval [0,1] and as a probability-space homology resulting from this decision-making process, maintains well-definedness when linearly combining the two subsets with coefficients 1 and ½, respectively. According to the developed model, such a high-dimensional deterministic function cannot be easily optimized.

Although VF has been rigorously verified for all required properties, such as well-definedness and one-to-one mapping, it will be demonstrated below that it cannot be optimized via traditional derivative-based methods.

$$\begin{aligned}
&VF = F + \frac{1}{2}G, \qquad 0 \le F \le \frac{1}{2}, \qquad \frac{1}{2} \le G \le 1 \\
&F = \frac{1}{11}B + \frac{3}{22}\log_K T + \frac{3}{11}P, \qquad K = \frac{4^e.PM}{PR} \\
&dF = \frac{3}{11}dP + \frac{1}{11}dB + \frac{3}{22}d(\log(T)_K), \\
&\frac{d}{dp}\left[P\left(pr(p)\right)\right] = \frac{d}{dp}\left(pr(p)\right) \times \frac{d}{dpr}\left(P\left(pr(p)\right)\right) \neq 0\ \forall p \\
&\frac{d}{d\left(pr(p)\right)}(P) = \frac{-pr(p)Ln\left(pr(p)\right) + 1 - pr(p)}{pr(p)} \\
&\frac{dB}{d\left(X(n)\right)} = (PM)^{X(n)} \times Ln(PM) \neq 0\ \forall X(n) \\
&\tfrac{d}{dt}\left[T\left(\tau(t)\right)\right] = \tfrac{d}{dt}\tau(t) \times \tfrac{d}{d(\tau)}T\left(\tau(t)\right),
\end{aligned} \tag{19}$$

$$\begin{aligned}
&\frac{d}{dt}\tau(t) = \begin{cases} not\ differentiable, & if\ t = 3k, \qquad k \in \mathbb{Z} \\ 0 & o.w \end{cases} \\
&\Rightarrow \frac{dT}{dt}\ is\ locally\ zero. \\
&H = \frac{1}{Ex(cl)^{CL(X)}} \\
&\frac{\partial H}{\partial Ex(cl)} = \frac{-CL(X)}{Ex(cl)^{CL(x)+1}}
\end{aligned}$$

$$\frac{\partial H}{\partial CL(X)} = \frac{1}{EX(cl)^{CL(X)}} \times Ln\big(Ex(cl)\big) \neq 0 \; \forall X$$

$$S = \log_2^5 \;, \frac{1}{S} = \log_5^2 \,, \quad H^{\frac{1}{S}} = \left(\frac{1}{Ex(cl)^{CL(X)}}\right)^{\log_5^2}$$

$$\frac{\partial G}{\partial Ex(cl)} = -\log_5^2 \; CL(X) \times \frac{1}{Ex(cl)^{CL(X)\log_5^2+1}} \neq 0, \qquad \forall X$$

$$\frac{\partial G}{\partial CL(X)} = \log_5^2 \times \left(\frac{1}{Ex(cl)^{CL(X)}}\right)^{\log_5^2} \; Ln(Ex(cl)) \neq 0 \; \forall X$$

Given this, the model continues to produce meaningful comparative outputs within the target interval [0,1), since it behaves as a probability-like function in which values closer to one indicate a higher replacement rate, representing a more profitable investment in each specific bidding competition. By adhering to the practical guidelines for each variable function, a simulation was conducted using random values within the domain of each variable.

4. RESULTS ANALYSIS AND CONCLUSION

The simulated values are presented in the table below.

Table 1. Simulated Values

	P	t	PM	BR	RP	K	Xn	cl	VF	EX
6173	0.001600	57	0.15	0.23	10	282.443872	22	5	0.238463	0.301709
247	0.015625	55	0.10	0.23	40	753.183660	49	2	0.251645	0.251189
8412	0.001600	26	0.05	0.23	20	188.295915	21	2	0.257091	0.251189
4601	0.001600	57	0.05	0.23	180	1694.663234	17	2	0.263232	0.251189
1161	0.001600	49	0.40	0.23	10	753.183660	26	1	0.271010	0.288540
...	...	...	...	...	...	...	...	...	...	...
3789	1.000000	3	1.00	0.23	1140	214657.342985	10	3	0.856156	0.519386
405	1.000000	10	1.00	0.23	1500	282443.872349	9	0	0.857047	0.588704
526	1.000000	8	1.00	0.23	910	171349.282558	20	0	0.864197	0.588704
7890	1.000000	1	0.70	0.23	1110	146305.925877	1	0	0.870715	0.588704
9520	1.000000	0	1.00	0.23	1010	190178.874048	39	4	0.871336	0.501187

10000 rows × 10 columns

For a more illustrative representation, the following scatter plot has been generated to show where the more profitable ranges are located and the corresponding values of their variables.

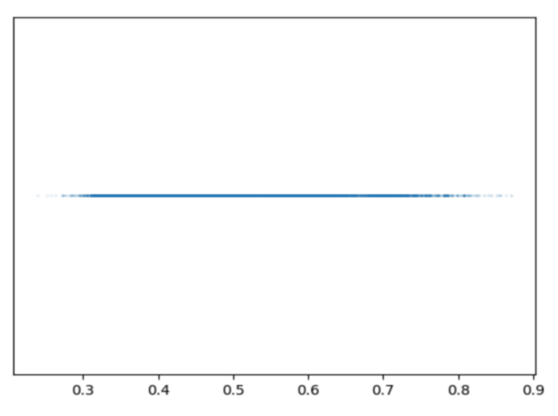
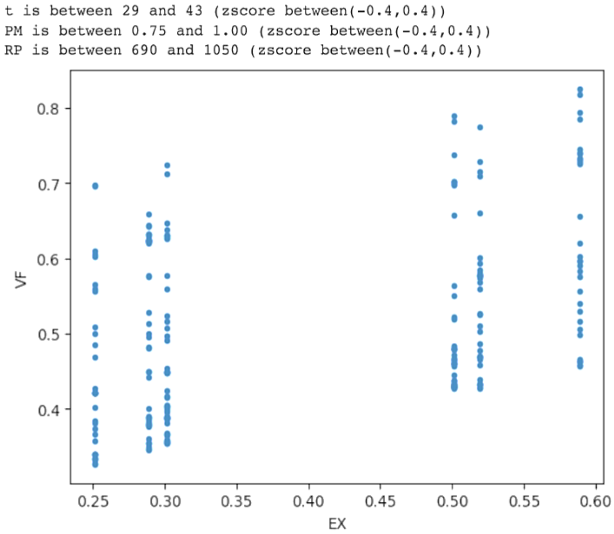


Fig. 1. The plot demonstrates a consistent logical correlation between the more favorable conditions of each variable and their positioning within profitable investment allocations.

Thus, the model preserves all the desirable constraints that were embedded in its construction. Most notably, the significant impact of claims and their associated costs on the decision-making process is clearly observable in the figure below.



```
t is between 29 and 43 (zscore between(-0.4,0.4))
PM is between 0.75 and 1.00 (zscore between(-0.4,0.4))
RP is between 690 and 1050 (zscore between(-0.4,0.4))
```

Fig. 2. While most of the variables and other influential elements of the bids remain nearly identical, differences in the values of EX() have a relatively significant impact on their positioning within the profitable spectrum.

Given that, both in the practical experience of investors in the construction sector and other participating contractors, as well as in academic literature—partially conducted at national or international scales—fair evaluation of bidding decisions is currently ambiguous, investment choices are often non-targeted. Nevertheless, any computational effort based on empirical criteria and numerical variables can provide insight and guide decision-making in such a complex and seemingly endless problem. The comparative tool that these efforts provide forms a logical foundation for such claims.

In this study, the primary focus was on constructing a function to develop a deterministic model that can be useful when evaluating a bid, deciding on participation, acceptance, and further investment in the proposed project. Based on methodological approaches, the following criteria were identified as key factors influencing investment in construction projects through a systematic literature review:

- Bid price function P(X)
- Project execution time T(X)
- Net project profit B(X)
- Probability of claims CL(X)
- Claim costs EX(CL)

These factors were carefully analyzed using differential and integral calculus and transformed into quantitative variables capable of numerical evaluation within a mathematical function. Subsequently, these criteria, informed by real-world contractor experience and supported by literature, were mapped into a quasi-probability space over the real interval [0,1). As noted, this mapping has the potential to serve as a homology to a probability function, reflecting its significant advantage in decision-making compared to traditional methods.

Another important aspect is the multivariable capability of this model. It simultaneously incorporates five distinct variables and establishes a logical relationship among them. The model was ultimately evaluated both for its mapping characteristics and its one-to-one property.

In summary, using this model, it is possible to predict, based on the characteristics and attributes of a bid—including estimated total price, construction duration, achievable net profit, and the potential occurrence of claims as a risk factor—the probability of success or failure in executing the project.

Declaration

We acknowledge that we used ChatGPT to enhance the academic writing of our manuscript while ensuring the originality and integrity of our work.

Transparency Statement

The data supporting this study are available upon reasonable request to the corresponding author, subject to ethical and confidentiality considerations.

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Declaration of Interest

The authors declare that they have no competing interests.

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