



Integration of Production Planning and Non-Cyclic Preventive Maintenance Scheduling for Multi-State Systems

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ARTICLE INFO	ABSTRACT
<p>Article History: Received 1 July 2024 Received in revised form 29 August 2024 Accepted 27 September 2024 Available online 28 September 2024</p>	<p>In various levels of manufacturing sectors, it is commonly assumed that preventive maintenance (PM) directly affects production scheduling and machine availability (i.e., reducing downtime). Surprisingly, however, the interaction between these factors is often overlooked. Maintenance planning is generally classified into cyclic and non-cyclic approaches. Among these, non-cyclic strategies provide more realistic and effective plans. The core principles of maintenance management advocate for implementing non-cyclic preventive actions in processes while minimizing corrective repairs and component replacements. This perspective simultaneously enables informed decision-making regarding both preventive maintenance and the corresponding production schedules. The integrated strategy of monitoring and preventive maintenance aims to fulfill production plans while minimizing the total associated costs including those related to maintenance (both preventive and corrective), setup, support, and production. This study proposes the integration of non-cyclic preventive maintenance scheduling with production planning in a multi-component system environment. An integrated model is developed to optimize decisions related to both maintenance and production scheduling. To efficiently solve the model and achieve high-quality solutions within a reasonable computation time, the Simulated Annealing (SA) metaheuristic algorithm is employed.</p>
<p>Keywords: Non-Cyclic Preventive Maintenance Planning; Production Planning and Scheduling; Metaheuristic Algorithms; Simulated Annealing Algorithm</p>	

1. INTRODUCTION

Since scheduling is a solution for the optimal utilization of available resources, determining the schedule, sequence, and prioritization of operations is one of the most crucial and influential factors in achieving success within manufacturing organizations. When implemented effectively, production scheduling leads to reduced waste, minimized machine downtime, timely responses to customer orders, just-in-time procurement of materials and equipment, and enhanced competitiveness. Efficient scheduling not only boosts production efficiency but also shortens task execution time, ultimately increasing a company's profitability.

In production organizations, productivity is defined as achieving the highest output with minimal time and effort. Accordingly, production scheduling aims to maximize efficiency and meet organizational objectives by allocating limited resources to a set of activities over a defined time horizon. However, scheduling models vary across

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manufacturing environments depending on their specific goals, priorities, and constraints. Therefore, selecting an appropriate scheduling model requires first identifying an organization’s objectives, priorities, and resource limitations.

Despite numerous studies on maintenance strategies, integrated models that simultaneously address production planning and maintenance remain relatively scarce. Most previous research has focused on maintenance planning through the lens of system reliability or has adopted production-inventory control models. These studies typically model machine failures using Markov chains with only two states operational and failed while assuming exponential lifetimes and constant failure rates. As a result, the critical aspect of preventive maintenance is often overlooked [1].

The integration of production planning and preventive maintenance in manufacturing systems is of considerable importance. In recent years, separate models for production planning and maintenance scheduling have been developed and analyzed. In this study, we propose a framework that integrates non-cyclic preventive maintenance with production planning in multi-component systems. The objective is to determine the optimal production quantities and maintenance schedules simultaneously. Numerical experiments are conducted to validate the model and assess the impact of incorporating maintenance into the production planning framework, particularly in terms of reducing total system costs.

The proposed model simultaneously determines the optimal timing for preventive maintenance and production decisions. It also specifies system-wide preventive maintenance strategies and production quantities to minimize total costs, including preventive and corrective maintenance, holding, backorders, setup, and production. Additionally, we compare the proposed model to previous frameworks that focused solely on production quantity decisions. Finally, a Simulated Annealing (SA) algorithm is employed to solve numerical instances of the problem efficiently. The following sections review the theoretical and empirical literature, followed by numerical examples to test and validate the proposed model.

2. LITERATURE REVIEW

This section reviews a number of previous studies relevant to the topic of this research, as summarized in the table below:

Table 1. Summary of Previous Research Findings

No.	Author(s)	Year	Key Findings
1	Su et al. [2]	2022	Multi-agent reinforcement learning-based approaches more effectively implement maintenance policies.
2	Akel et al. [3]	2022	Demonstrated the ability to reduce total maintenance costs by 5.6%.
3	Zhen et al. [4]	2021	Presented an approach that optimizes PM intervals by integrating risk and cost for critical safety barriers in offshore oil facilities, enabling more accurate PM implementation.
4	Duarte et al. [5]	2020	Modeled preventive maintenance scheduling of power generation units as a nonlinear stochastic optimization problem, effectively solved using PSO and GA algorithms.
5	Miyata et al. [6]	2019	Computational results showed that their proposed PM scheduling method outperforms others across large problem sizes.
6	Zaidan et al. [7]	2019	The proposed model delivered higher-quality solutions compared to genetic algorithms and particle swarm optimization.
7	Chansompat et al. [8]	2019	Conducted a case study in a manufacturing plant, revealing a 63% reduction in total production costs.
8	Abubakar et al. [9]	2016	Proposed a modified simulated annealing algorithm for integrated production planning, outperforming harmony search in solution quality.
9	Chakraborty et al. [10]	2015	Developed a PSO-based model for integrated production planning, with results outperforming genetic and fuzzy genetic algorithms.

3. RESEARCH MODEL AND ASSUMPTIONS

In this study, we consider a manufacturing system composed of a set of machines referred to as components, which are arranged within the system based on optimal layout configurations. We examine a multi-state system that can incorporate various component configurations, including series, parallel, and series-parallel arrangements (Weinstein & Chung, 1999; Nourelfath et al., 2010; Tong, 2004; Raza et al., 2007) [11–14].

We assume that each component has two possible states: operational or failed. The state of the j -th component is characterized by its nominal performance rate G_j . Accordingly, the production system is treated as a multi-state system with limited production rates denoted by g_k , where g_k represents the number of products produced per unit time. Each product is assumed to require the same amount of processing time.

3.1. Problem Assumptions

The model is based on the following assumptions:

- Components are economically and structurally independent. This implies that the total maintenance cost of a group of components equals the sum of the individual maintenance costs. Moreover, the condition of any given component does not influence the lifetime distribution of others.
- Preventive maintenance is carried out under a full maintenance policy.
- Minor repairs do not affect the remaining lifetime of components.
- All products require an equal amount of work, proportional to system performance.
- There is no initial inventory or backordered demand at the beginning of the planning horizon.

3.2. Symbols and Parameters

A. Indices

- j : Index for components, ($1 \leq j \leq n$)
- k : Index for system states, ($1 \leq k \leq K$)
- K : Number of production rates
- τ : Length of the preventive maintenance planning horizon
- n : Number of components
- P : Set of products
- p : Product, where $p \in P$
- S : Number of sub-periods within interval L
- s : Index for maintenance planning periods, ($1 \leq s \leq S$)
- T : Number of production planning periods
- t : Index for production planning time periods, ($1 \leq t \leq T$)

B. Parameters

ΔE : Energy loss in the simulated annealing algorithm

- δ_j : Degradation factor of component j when PM actions are performed at the start of production planning periods
- AG : $n \times TS$ matrix representing the effective age of component j at the beginning of each maintenance planning period
- A : $n \times T$ matrix indicating the availability of component j during each production planning period t

- CMR: Diagonal $n \times n$ matrix of minimum corrective maintenance costs; CMR_{jj} ($j=1, \dots, n$) denotes the minimum repair cost for component j
- CPR: Diagonal $n \times n$ matrix of preventive replacement costs; CPR_{jj} ($j=1, \dots, n$) denotes the preventive replacement cost for component j
- MMM: $n \times TS$ matrix indicating the expected number of failures for component j during each maintenance period
- P_{jt} : Probability distribution of component j during production period t
- QQQ: Diagonal $TS \times T$ scaling matrix
- RRR: Diagonal $n \times n$ cost reduction matrix if PM is performed at the start of the production planning period
- TMR: Diagonal $n \times n$ matrix of minimum repair times; TM_{jj} ($j=1, \dots, n$) is the minimum repair time for component j
- TPR: Diagonal $n \times n$ matrix of preventive replacement times; TPR_{jj} ($j=1, \dots, n$) denotes the preventive replacement time for component j
- π_{pt} : Unit production cost for product p during period t
- τ^{ts} : Maintenance planning period sss within production period t
- A_j^{ts} : Availability of component j during maintenance time τ^{ts}
- a_j^{ts} : Age function of component j at the end of maintenance time τ^{ts}
- b_{pt} : Penalty cost per unit for unmet demand (e.g., lost sales) of product p at the end of period t
- C : Cooling rate constant in the simulated annealing algorithm
- CM: Total maintenance cost
- CT: Total cost of maintenance and production
- d_{pt} : Demand for product p to be met by the end of period t
- EEE: Objective function energy in the optimization algorithm
- $f_j(0)$: Initial lifetime function of component j
- G_j : Nominal production rate of component j
- g_k : Production rate for system state kkk
- G_{MSS}^t : Available production capacity of the multi-state system (MSS) during planning period t
- HHH: Planning horizon
- h_{pt} : Inventory holding cost per unit of product p at the end of period t
- L : Length of production planning periods
- $M_j(t)$: Expected number of failures/repairs of component j within interval $[0, t]$
- M_j^{ts} : Expected number of failures of component j during maintenance period τ^{ts} ($j = 1, \dots, n, t = 1, \dots, T, s = 1, \dots, S$)
- N : Number of possible preventive maintenance policy matrix combinations Z
- $prob_k$: Probability of system being in state $k, (1 \leq k \leq K)$
- q_{ts}^i : Binary indicator variable; equals 1 if $t=i$, 0 otherwise
- $r_j(0)$: Initial linear degradation function of component j
- Set_{pt} : Fixed setup cost for production of product p in period t
- T_{max} : Maximum temperature in the simulated annealing algorithm
- T_{min} : Minimum temperature in the simulated annealing algorithm
- www : Random value drawn from the interval $[0, 1]$
- z_j^{ts} : Binary decision variable; equals 1 if PM is performed on component j at maintenance time τ^{ts} , 0 otherwise

Decision Variables

- Z : Binary matrix representing the system's preventive replacement policy
- B_{pt} : Backorder level of product p at the end of period t
- I_{pt} : Inventory level of product p at the end of period t
- x_{pt} : Quantity of product p to be produced in period t
- y_{pt} : Binary variable; equals 1 if production of product p is initiated in period t , 0 otherwise

3.3. Mathematical Modeling

Objective Function:

$\text{Minimize CT} = \sum_{p \in P} \sum_{t=1}^T (h_{pt}I_{pt} + b_{pt}B_{pt} + \pi_{pt}X_{pt} + set_{pt}y_{pt}) + CM(Z)$	(1)
Subject to:	
$X_{pt} - I_{pt} + I_{p(t-1)} + B_{pt} - B_{p(t-1)} = d_{pt}, \quad p \in P \text{ and } t = 1 \dots T$	(2)
$X_{pt} \leq \left(\sum_{q \geq t} d_{pq} \right) y_{pt}, \quad p \in P \text{ and } t = 1 \dots T$	(3)
$\sum_{p \in P} X_{pt} \leq G_{MSS}^t L, \quad t = 1 \dots T$	(4)
$x_{pt}, I_{pt} \text{ and } B_{pt} \text{ integers, } p \in P, t = 1, \dots, T$	(5)
$y_{pt} \text{ Binary } p \in P, t = 1, \dots, T, \text{ and } Z \text{ Binary matrix}$	(6)
$B_{p0} = 0, I_{p0} = 0, p \in P$	(7)

As shown in Equation (1), the objective function aims to minimize the total cost, which comprises the sum of inventory holding costs, backorder costs, total production costs, total setup costs, and total maintenance costs $CM(Z)$.

Constraint (2) relates the inventory level or backorders at the beginning or end of period t to the production and demand within that period. Since the objective function can be improved by reducing I_{pt} and B_{pt} , no optimal solution exists where both $I_{pt} > 0$ and $B_{pt} > 0$ simultaneously.

Equation (2-3) ensures that the sum of inventory (or backorders) for product p at the end of period t equals the inventory (or backorders) from the previous period plus the total production of that product in the current period, minus the demand for that period.

Constraint (3) ensures that if $y_{pt} = 0$, then $x_{pt} = 0$; and if $y_{pt} = 1$, then $x_{pt} \geq 0$. In Equation (3), the term $\sum_{q \geq t} d_{pq}$ serves as an upper bound for x_{pt} .

Equation (4) corresponds to the available production capacity constraint.

Finally, Constraints (5) and (6) specify the non-negativity and binary nature of the decision variables, respectively. Constraint (7) defines the initial conditions for inventory levels and backorders.

4. SOLUTION METHODS

4.1. Exhaustive Search Method

The exhaustive search method was employed in the study by Nouralfs and colleagues (2010). This approach involves enumerating all possible combinations of the preventive maintenance policy matrix (Z), calculating their associated parameters namely, $CM(Z)$ and G_{MSS}^t and ultimately solving all possible lot-sizing problems to determine the optimal solution.

The number of lot-sizing problems to be solved, denoted by NNN, corresponds to the total number of possible configurations of the maintenance policy matrix (Z). This number depends on the total number of planning periods (T), sub-periods (SSS), and components (nnn), and is given by the following equation:

$$N = 2^{n(ST-1)} \tag{8}$$

The principal advantage of this method lies in its guarantee of reaching the optimal solution. However, it is only applicable to small-scale problems. In fact, any increase in n , S , or T leads to an exponential growth in the number of possible combinations of the maintenance policy matrix Z , making the method computationally infeasible for large instances.

4.2. Simulated Annealing (SA) Algorithm

The Simulated Annealing (SA) algorithm was first introduced in the early 1980s by Kirkpatrick and colleagues. In the context of the current problem, the initial solution used in the optimization process assumes no preventive maintenance policies are in place. As illustrated in Figure 2, a new solution is generated by randomly modifying the current one. This is done by selecting one or more elements from the maintenance policy matrix Z at random and toggling their values from 0 to 1, or vice versa.

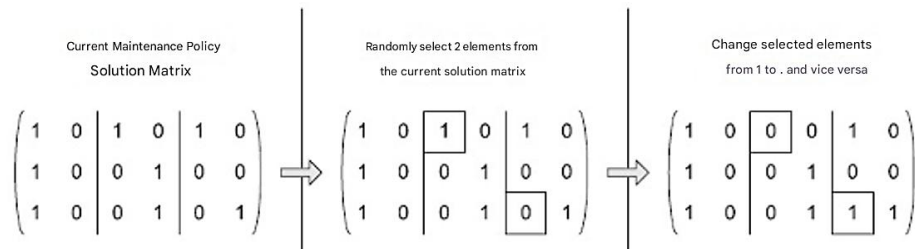


Fig. 1. Sample movement region for a production system with $n = 3$, $T = 3$, and $S = 2$

Accordingly, the new value of the objective function (i.e., total production and maintenance costs) is calculated and compared with the previous objective function based on the energy criteria defined by the simulated annealing (SA) method. The temperature decreases from T_{max} to T_{min} following the cooling schedule $T_{m+1} = T_m C$, where C is a constant.

The SA algorithm continues to run until the temperature reaches T_{min} . To tune the algorithm’s parameters, different levels are considered, and the optimal level for each parameter is determined using the Taguchi method. The parameters of the simulated annealing algorithm along with their corresponding levels are presented in Table 1.

For each configuration of the algorithm, an experimental design is created using Minitab software. According to Table 2, the simulated annealing algorithm involves four factors, each at three levels, resulting in a total of nine experimental runs. It is worth mentioning that the Relative Percentage Deviation (RPD) chart for the simulated annealing algorithm is illustrated in Figure 2.

Table 2. Parameter Settings for the Simulated Annealing (SA) Algorithm

Parameter	Level 1	Level 2	Level 3
Max it	5000	6000	7000
TOT_0T0 (Initial Temperature)	500	600	700
Alpha (Cooling Rate)	0.93	0.97	0.99
Type of Perturbation Mechanism (Pm)	Swap (1)	Inversion (2)	Remove-and-Reinsert (3)

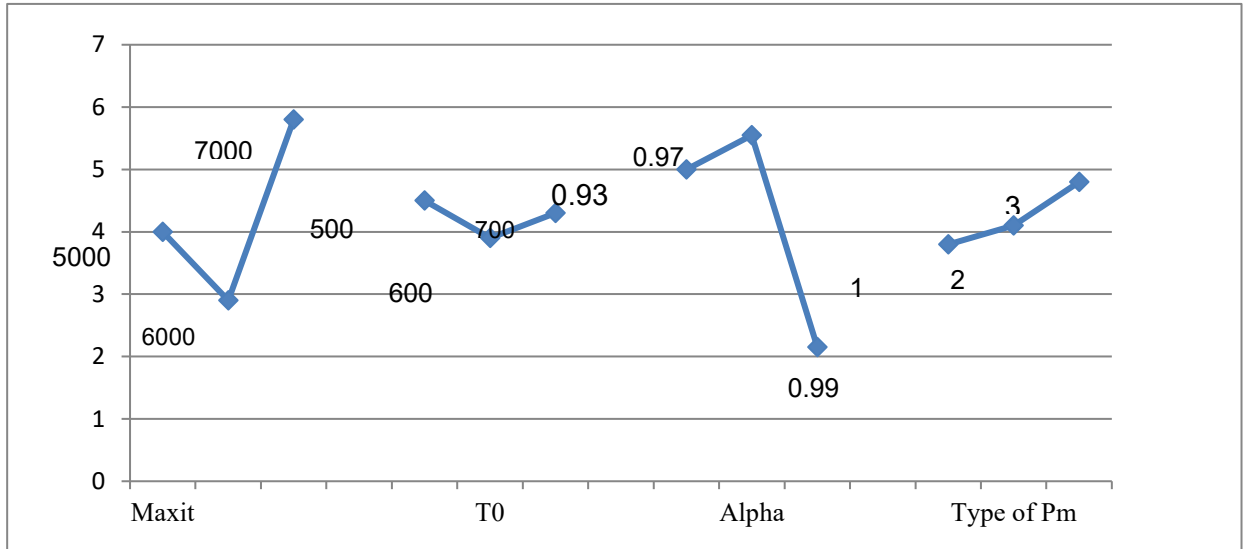


Fig. 2. RPD Chart of the Simulated Annealing (SA) Algorithm

As illustrated in the chart above, a lower mean value indicates better performance. According to the RPD chart, the simulated annealing algorithm achieves its best performance with the following parameter settings: Max it = 6000, T0=600, Alpha = 0.99, and Type of Pm = Swap. These values are employed for testing the problem scenarios.

5. NUMERICAL ANALYSIS

In this study, both categories of solution approaches deterministic and stochastic are employed to solve the problem. The Exhaustive Search (ES) method is used as the deterministic approach, while the Simulated Annealing (SA) algorithm serves as the metaheuristic (probabilistic) method.

5.1. Evaluation of Small-Scale Problems

As shown in Figure 3, a sample problem is considered based on a multi-state system comprising three two-state components arranged in a series-parallel configuration. The lifetime distribution and characteristics of each component are presented in Table 3. The monthly demand for each product is provided in Table 4, and Table 5 outlines the inventory holding cost, backorder cost, setup cost, and production cost for each product. These cost values remain constant across all time periods.

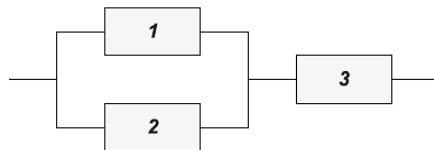


Fig. 3. A Series-Parallel System with Three Two-State Components

Table 3. Lifetime Distribution and Reliability Parameters of Components

Component j	G _j (items/month)	CPR _j (\$)	CMR _j (\$)	TPR _j (weeks)	TMR _j (weeks)	Lifetime Distribution
1	105	5000	200	0.09	0.05	Weibull (3.3)
2	110	4000	300	0.08	0.04	Weibull (2.2)
3	205	3000	250	0.09	0.02	Weibull (2.2)

Where: CPR = Cost of Preventive Replacement; CMR = Cost of Minimal Repair; TPR = Time of Preventive Replacement; TMR = Time of Minimal Repair

Table 4. Monthly Demand for Each Product

Period t	Demand for Product 1 d _{1t} (items)	Demand for Product 2 d _{2t} (items)
1	95	95
2	95	95
3	95	90
4	90	90

Solving the problem using the exhaustive search method required approximately 2 hours and 30 minutes. The total number of maintenance policy matrices Z generated based on Equation (8), $N = 2^{21} = 2\,097\,152$, results in 2,097,152 combinations. The optimal solution, which minimizes both the total maintenance cost CM and the overall production and maintenance costs, is presented in Table 5 along with the associated maintenance policies and system capacities.

Table 5. Product Cost Parameters

Product p	Holding Cost h _{pt} (USD)	Setup Cost b _{pt} (USD)	Backorder Cost s _{pt} (USD)	Production Cost π _{pt} (USD)
1	40	150	1000	100
2	40	150	1000	100

Table 6. Maintenance Policy Matrix

Component j	Maintenance Policy for Minimizing Total Production Planning Cost				Maintenance Policy for Minimizing Maintenance Cost During Production Planning Period			
	T ₁	T ₂	T ₃	T ₄	T ₁	T ₂	T ₃	T ₄
1	1	0	1	0	0	1	0	0
2	1	0	0	0	1	0	0	0
3	1	0	0	0	1	0	0	0
G ^t _{MSS}	193.52	187.52	188.85	182.88	193.52	171.22	193.52	171.22

The maintenance policy that minimizes only maintenance costs results in a periodic strategy for all components, yielding a total cost of \$136,255. However, the cost-optimal policy, which minimizes the overall production and maintenance cost, proposes a cyclic strategy for component 1 and results in a lower total cost of \$132,894. This implies a 2.5% improvement due to the integration of production planning with preventive maintenance scheduling.

The corresponding production plans for each strategy are presented in Tables 7 and 8, respectively. As shown in Table 7, the 2.5% improvement can be attributed to the enhanced capacity utilization under the cost-optimal policy. According to Table 6, the maintenance-only optimization leads to substantial backorders, indicating an overemphasis on minimizing maintenance cost at the expense of production feasibility.

By modifying the maintenance strategy for component 1, the integrated approach increases the Maximum System Supply (MSS) capacity, allowing for better alignment between demand and production capability. This highlights the benefit of integrated planning over isolated maintenance optimization.

Table 7. Production Plan for Maintenance Cost Minimization

Period	A - Production	A - Inventory	A - Backorder	A - Setup	B - Production	B - Inventory	B - Backorder	B - Setup
1	95	0	0	1	98	3	0	1
2	79	0	16	1	92	0	0	1
3	111	0	0	1	82	0	8	1
4	73	0	17	1	98	0	0	1

Table 8. Production Plan for Total Cost Minimization

Period	A - Production	A - Inventory	A - Backorder	A - Setup	B - Production	B - Inventory	B - Backorder	B - Setup
1	98	3	0	1	95	0	0	1
2	92	0	0	1	95	0	0	1
3	95	0	0	1	90	0	0	1
4	90	0	0	1	90	0	0	1

5.2. Analysis of Large-Scale Problems

Considering that the previous numerical example required approximately 2 hours and 30 minutes to solve a joint capacitated lot-sizing problem with a solution space size of $2^{21} = 2,097,152$, the enhanced Simulated Annealing (SA) algorithm, as developed in the previous section, is utilized as an alternative approach. This algorithm is designed to find a feasible solution either optimal or near-optimal within a reasonable computational time for large-scale problems.

To demonstrate this, a system comprising 10 components ($n = 10$) is considered. These components are arranged in three subsystems (1, 2, and 3) that are connected in series, as illustrated in Figure 4. The characteristics and lifetime distribution of each component are provided in Table 9, while the periodical demands and costs are detailed in Tables 10 and 11.

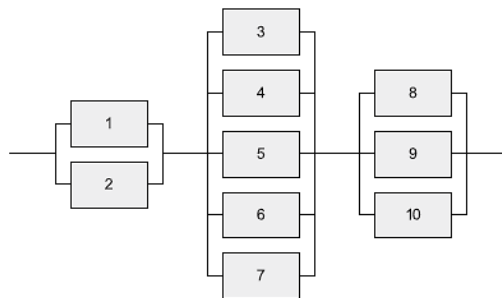


Fig. 4. Series–Parallel Configuration of the MSS Components

Table 9. Characteristics of Components (n = 10, T = 5, S = 1)

Component (j)	G_k (Items/Month)	$CPR_j(S)$ (USD)	$CMR_j(S)$ (USD)	TPR_j (Months)	TMR_j (Months)	Lifetime Distribution
1	100	1500	1000	0.03	0.06	Weibull (2.2)
2	50	1500	1200	0.03	0.08	Weibull (3.3)
3	100	2000	1400	0.04	0.10	Weibull (2.2)
4	50	2000	1600	0.02	0.10	Weibull (3.3)
5	50	3000	1800	0.05	0.12	Weibull (2.2)
6	50	2500	2000	0.06	0.14	Weibull (3.3)
7	50	3000	2200	0.05	0.15	Weibull (2.2)
8	50	3500	2400	0.04	0.13	Weibull (3.3)
9	150	4000	2600	0.03	0.10	Weibull (2.2)
10	100	4500	2800	0.01	0.09	Weibull (3.3)

Legend:

- G_k : Generation rate
- CPR_j : Cost of preventive replacement
- CMR_j : Cost of maintenance replacement
- TPR_j : Threshold for preventive replacement
- TMR_j : Threshold for maintenance replacement

Table 10. Product Demand for n = 10, T = 5, and S = 1

Period t	Demand of Product 1 d_{1t} (Items)	Demand of Product 2 d_{2t} (Items)
1	50	50
2	10	10
3	50	50
4	70	70
5	150	150

Table 11. Product Cost Data for n = 10, T = 5, and S = 1

Product P	Holding Cost $h_{pt}(S)$	Backorder Cost $b_{pt}(S)$	Setup Cost $s_{pt}(S)$	Production Cost $\pi_{pt}(S)$
1	40	250	500	100
2	40	250	500	100

The Simulated Annealing (SA) algorithm was employed to solve this instance. The cooling process followed the rule $T_{m+1} = T_m C$, where $C=0.999$, starting from $T_{max} = 100$ and continuing until $T_{min} = 0.1$. New solutions were generated by modifying two randomly selected elements in the maintenance policy matrix. The numerical experiment was repeated 100 times, and in 94% of the runs, the same optimal solution was obtained.

As shown in Table 12, the integrated production and maintenance planning approach yielded the best solutions. The results suggest that preventive replacement (PR) should be performed at the beginning of the fourth maintenance planning period for all components. The total cost of the optimal solution was \$170,612, with a standard deviation of approximately 31 and a coefficient of variation less than 0.02%. This solution was obtained in an average computational time of about 34 seconds.

Obtaining any feasible solution using exhaustive search methods with current technology would be infeasible. The production schedule corresponding to the optimal solution is presented in Table 13. Overall, the SA method consistently produced high-quality solutions with minimal deviation and within a short processing time.

6. CONCLUSION AND SUGGESTIONS

Table 12. Maintenance Policy for the Best Solution Obtained by SA Algorithm

Component j	T ₅	T ₄	T ₃	T ₂	T ₁
1	1	0	0	1	0
2	1	0	0	1	0
3	1	0	0	1	0
4	1	0	0	1	0
5	1	0	0	1	0
6	1	0	0	1	0
7	1	0	0	1	0
8	1	0	0	1	0
9	1	0	0	1	0
10	1	0	0	1	0

Table 13. Optimal Production Plan Obtained by the SA Algorithm

Period	Product A				Product B			
	Production	Inventory	Backorder	Setup	Production	Inventory	Backorder	Setup
1	60	10	0	1	60	10	0	1
2	0	0	0	0	0	0	0	0
3	50	0	0	1	50	0	0	1
4	70	0	0	1	135	65	0	1
5	150	0	0	1	85	0	0	1

7. CONCLUSION AND FUTURE WORK

This study aimed to address key challenges in production systems by proposing an integrated planning model that coordinates both production scheduling and preventive maintenance activities. The importance of structured and scientifically designed maintenance processes in ensuring the operational reliability of production systems cannot be overstated.

A unified model for general preventive maintenance scheduling in multi-state systems (MSS) was developed. On the production side, the model considers inventory levels, backorders, setup times, and production quantities for each product in every planning period. From the maintenance perspective, it determines the scheduling of preventive maintenance actions that can be conducted during production periods for each component.

The model relies on a matrix-based formulation that incorporates key system parameters such as availability and overall capacity. Two solution methods were applied: Exhaustive Search (ES) and Simulated Annealing (SA). While the ES method guarantees an optimal solution, its computational complexity restricts its application to small-scale systems. The SA algorithm, on the other hand, efficiently produces high-quality solutions within reasonable time frames.

The findings clearly demonstrate that integrating production planning with cyclical maintenance significantly reduces total costs. Moreover, the proposed model enhances system responsiveness by improving coordination between demand requirements and available capacity.

Declaration

We acknowledge that we used ChatGPT to enhance the academic writing of our manuscript while ensuring the originality and integrity of our work.

Transparency Statement

The data supporting this study are available upon reasonable request to the corresponding author, subject to ethical and confidentiality considerations.

Acknowledgments

We would like to express our gratitude to all individuals who contributed to this project.

Declaration of Interest

The authors declare that they have no competing interests.

Funding

This research received no specific grant from any funding agency, commercial, or not-for-profit sectors.

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